Robust Design of Thermopiezoelectric Actuators

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Introduction

THE theory of thermopiezoelectricity can be applied to piezoelectric materials that are utilized in the environments where thermal effects are important.[1] The adverse environments where the control systems are functioning may combine multiple fields, which must be considered for an effective and reliable control. The theory of thermopiezoelectricity has been applied to system control only in the past few years.[2]

The purpose of the robust design of a system is to ensure its reliable and steady performance despite the changes in parameters and existence of disturbances. The Taguchi robust design methodology was developed by Taguchi[3] in the 1950s and 1960s to meet the demand for high-performance products and systems. The methodology has been tested in vastly different fields, and its value has been proven through applications.

In this work, the robustness of a thermopiezoelectric actuator pair bonded to a cantilever beam is studied using thermopiezoelectric finite element equations in Taguchi methodology. The objective of the robustness study is to identify the parameters and the levels of their effects on the closed-loop system performance.

Modeling of Thermopiezoelectric Continua

The linear quasistatic equations for a thermopiezoelectric continuum are given by[4]

$$\begin{gathered} \boldsymbol{T} = c\boldsymbol{S} - e\boldsymbol{E} - \lambda\theta, \qquad \boldsymbol{D} = e^T\boldsymbol{S} + \varepsilon\boldsymbol{E} + \boldsymbol{P}\theta \\ \eta = \lambda^T\boldsymbol{S} + \boldsymbol{P}^T\boldsymbol{E} + \alpha\theta \end{gathered} \tag{1}$$

where $\boldsymbol{T}$, $\boldsymbol{S}$, $\boldsymbol{D}$, and $\boldsymbol{E}$ are the vectors of stress, strain, electrical displacement, and electrical field, respectively. In Eq. (1) η and θ are the entropy per unit volume and temperature variation relative to a reference temperature Θ_0, respectively. Other symbols in the equation stand for the constitutive coefficients with appropriate dimensions.[2] The generalized heat equation is written as[5]

$$\Theta\dot{\eta} = -\nabla^T\boldsymbol{h} + W \tag{2}$$

where Θ is the absolute temperature ($\Theta = \Theta_0 + \theta$), $\boldsymbol{h}$ is the vector of heat flux, and W is the heat source intensity per unit volume. The following relations for mechanical, electrical, and thermal fields are also noted:

$$\boldsymbol{S} = L_u\boldsymbol{u}, \qquad \boldsymbol{E} = -\nabla\phi, \qquad \boldsymbol{h} = -K\nabla\theta \tag{3}$$

where $\boldsymbol{u}$ is the displacement vector, ϕ is the electrical potential, and K and L_u are the matrices of heat conduction and differential operator, respectively. The use of the third equation of Eq. (1) and Eq. (3) in Eq. (2), together with the finite element approximations, yields one of the global finite element equations for the thermopiezoelectric continuum as

$$-C_{\theta u}\dot{\boldsymbol{u}} + C_{\theta\phi}\dot{\phi} - C_{\theta\theta}\dot{\theta} - K_{\theta\theta}\theta = \boldsymbol{Q} \tag{4}$$

where $\boldsymbol{Q}$ is the external heat vector and $C_{\theta u}$, $C_{\theta\phi}$, $C_{\theta\theta}$, and $K_{\theta\theta}$ are the finite element matrices.[2] To derive the other two equations for a unique solution in $\boldsymbol{u}$, ϕ, and θ, a functional Π is defined as[5]

$$\begin{aligned} \Pi = {} & \int_V (G + \eta\Theta)\,\mathrm{d}V - \int_V \boldsymbol{u}^T\boldsymbol{P}_b\,\mathrm{d}V - \int_{S_1} \boldsymbol{u}^T\boldsymbol{P}_s\,\mathrm{d}S \\ & - \boldsymbol{u}^T\boldsymbol{P}_c + \int_{S_2} \phi\sigma\,\mathrm{d}S \end{aligned} \tag{5}$$

where G is the thermodynamic potential; $\boldsymbol{P}_b$, $\boldsymbol{P}_s$, and $\boldsymbol{P}_c$ are the vectors of body, surface, and concentrated forces, respectively; and σ is the surface charge. Hamilton's principle is expressed as

$$\delta\int_{t_1}^{t_2} (K_i - \Pi)\,\mathrm{d}t = 0 \tag{6}$$

where K_i is the kinetic energy given by

$$K_i = \frac{1}{2}\int_V \rho\dot{\boldsymbol{u}}^T\dot{\boldsymbol{u}}\,\mathrm{d}V \tag{7}$$

Using Eqs. (1), (5), and (7) with the finite element approximations in Eq. (6) yields the other two global finite element equations for the thermopiezoelectric continuum as

$$\begin{gathered} M_{uu}\ddot{\boldsymbol{u}} + K_{uu}\boldsymbol{u} + K_{u\phi}\phi - K_{u\theta}\theta = \boldsymbol{F} \\ K_{\phi u}\boldsymbol{u} - K_{\phi\phi}\phi + K_{\phi\theta}\theta = \boldsymbol{G} \end{gathered} \tag{8}$$

where $\boldsymbol{F}$ and $\boldsymbol{G}$ are the global vectors for applied force and charge, respectively, and M_{uu}, K_{uu}, $K_{u\phi}$, $K_{u\theta}$, $K_{\phi\phi}$, and $K_{\phi\theta}$ are the finite element matrices.[2]

Taguchi Robust Design Methodology

Taguchi robust design methodology was developed to ensure the consistent performance of a system despite the variances in parameters of the system and other noise factors. Taguchi methodology is composed of three design steps, namely, system design, parameter design, and tolerance design steps.[3] The system design step is the step where the conceptual design takes place for the initial system design. The parameter design phase is the phase where it is attempted to make the system robust against noise factors including variations in design and manufacturing of the system and external disturbances. The tolerance design step is the final step, which is implemented to improve the system performance if the parameter design stage is not satisfactory.

The application of Taguchi methodology to a cantilever beam structure with a thermopiezoelectric actuator pair is implemented as follows:

1) An objective function f and several parameters affecting the objective function are identified. The objective function is chosen as the absolute sum of the vertical displacements (deflections) of the tip of the beam subjected to simultaneous mechanical and thermal loads. Various parameters that influence the objective function are identified as the length and thickness of the actuators and the distance of the actuators from the fixed end of the beam.

2) Levels are selected for the parameters to indicate their limits of variation. These levels are comparable with the ranges commonly used in the literature.

3) An appropriate orthogonal array is chosen. The choice of the orthogonal array depends on the parameter levels. L_9 orthogonal array is used in this study.[3]

4) The response of the control system due to mechanical and thermal loads are found using the parameter settings in the orthogonal arrays, and the values of the objective function and signal-to-noise ratio (S/N) for the smaller-the-better characteristic are computed. The best parameter setting is decided to occur at the highest value of S/N.

5) An analysis of variance test is performed to calculate the percentage effects of the parameters on the objective function.

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Table 1 Levels of parameters

No.	B_{ss}, m	Z, m	d, m
1	0.0002	0.05	0.20
2	0.0005	0.12	0.23
3	0.0008	0.20	0.26

Table 2 Temperature increases with changes in thickness of actuators

B_{ss}, m	θ_1, K	θ_2, K	θ_3, K	θ_4, K
0.0002	5.6	5.5	7.4	5.3
0.0005	6.5	6.3	10.0	6.0
0.0008	6.3	6.3	8.9	5.0

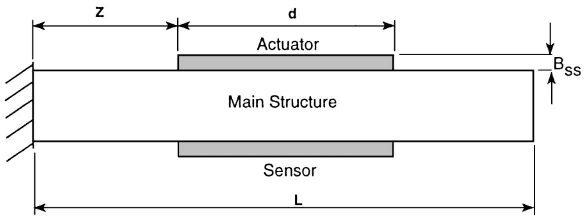

Fig. 1 Cantilever beam with thermopiezoelectric actuator pair.

Case Study

A cantilever beam mounted with a thermopiezoelectric actuator pair, shown in Fig. 1, is considered in the case study. The system is divided into 25 elements for the finite element modeling. The structure and actuators are made up of aluminum and PVDF, respectively, whose material properties are given in Ref. 2. The length L of the structure and its height are fixed at 0.5 m and 0.01 m, respectively. The thickness and length of the actuators, B_{ss} and d, and distance of the actuators from the fixed end of the beam, Z, are chosen to be the parameters, which vary in certain limits (Table 1) and influence the control performance.

The control performance is realized by the objective function as defined before. The mechanical load is assumed to be a unit step force acting downward at the tip of the beam. While the mechanical load is acting, the system is assumed to be equilibrated at certain temperatures due to steady-state thermal fields of two types. In the first type, a uniform temperature increase of 25 K for the whole system is assumed. A heat flux of 500 W/m^2 directed upward is imposed on the system as the second type. Because the system is much larger in size in the longitudinal direction, the temperature variations due to the heat flux take place mostly in the vertical direction. Hence, four distinct temperature increases are noted. These are θ_1, θ_2, θ_3, and θ_4, which represent the temperature increases along the bottom and top sides of the beam and along the bottom and top sides of the bottom and top actuators, respectively. The computed temperature increases with the variation in thickness of the actuators are given in Table 2. Note that in the tables, the subscripts 1 and 2 on f, S/N, and percentage effect refer to the simulations where the thermal fields of the first and second type are imposed, in addition to the mechanical load. The beam is controlled by the linear quadratic regulator control technique[6] using thermopiezoelectric actuators.

Taguchi methodology is performed using L_9 orthogonal array. The L_9 orthogonal array, the values of f, S/N, and percentage effect are given in Tables 3 and 4. The parameter settings at which the S/N attains the highest and lowest values are named as the best and worst designs, respectively. It is clear from Table 3 that the best design is achieved at the highest levels of B_{ss} and d and at the lowest level of Z for the thermal fields of both types. On the other hand, the worst design corresponds to the parameter setting 8 for the first thermal field and setting 3 for the second. The superiority of the best design over the two worst designs is evident in the closed-loop system deflection plot shown in Fig. 2 for the thermal field of the second type.

Table 3 L_9 orthogonal array, objective function, and S/N values

No.	B_{ss}	Z	d	$B_{ss} \times Z$	f_1	$(S/N)_1$	f_2	$(S/N)_2$
1	1	1	1	1	0.669	3.48	0.671	3.47
2	1	2	2	2	0.674	3.43	0.672	3.34
3	1	3	3	3	0.675	3.41	0.683	3.31
4	2	1	2	3	0.653	3.70	0.557	5.08
5	2	2	3	1	0.666	3.53	0.629	4.02
6	2	3	1	2	0.686	3.27	0.616	4.21
7	3	1	3	2	0.632	3.99	0.361	8.85
8	3	2	1	3	0.697	3.13	0.446	7.02
9	3	3	2	1	0.696	3.14	0.538	5.37

Table 4 Percentage effects on objective function

Parameters and interactions	Percentage effect$_1$	Percentage effect$_2$
B_{ss}	Not significant	69
Z	52	Not significant
d	25	Not significant
$B_{ss} \times Z$	Not significant	Not significant

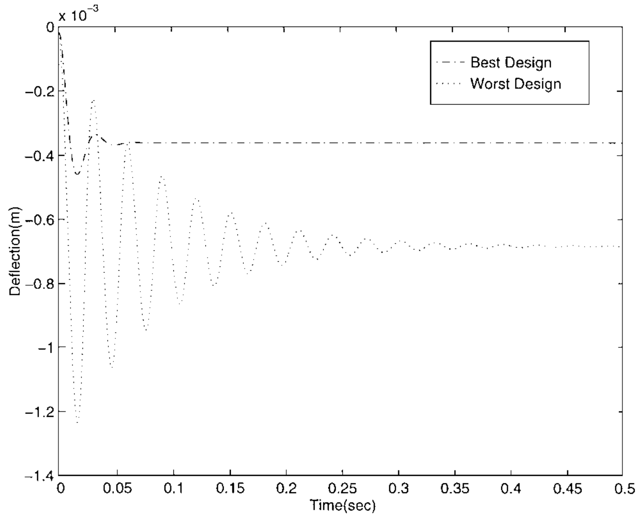

Fig. 2 Tip deflection of closed-loop system due to mechanical and second type thermal loads.

Conclusions

Taguchi robust design methodology is applied to a closed-loop structure containing a cantilever beam and a thermopiezoelectric actuator pair. The control performance of the system is measured through an objective function taken as the absolute sum of the tip deflections of the system subjected to mechanical and thermal disturbances. The parameters affecting the control performance are chosen as the length, thickness, and location of the actuator pair. Based on the numerical results, it is concluded, in general, that the actuators that are large in size and close to the fixed end perform better in controlling the structural behavior. The degree of importance of the parameters varies depending on the type of thermal disturbance.

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Lateral Directional Aircraft Control Using Eigenstructure Assignment

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Introduction

EIGENSTRUCTURE assignment has received considerable attention as a method for design of flight control systems.[1–3] Although recently overshadowed by various H_∞-based techniques in the research literature, eigenstructure techniques offer a simple method for designing low-order, easily implementable controllers that can directly incorporate handling quality specifications. It is well known that, for a controllable linear system, eigenstructure techniques can be used to design a linear feedback controller that places closed-loop eigenvalues and shapes closed-loop eigenvectors into desired configurations.[4] The question for the designer is how to specify these desired eigenvalues and eigenvectors. The approach developed in this Note is to use a state-space model of desired responses that is based on flying quality specifications. This state-space model is then used to generate the desired eigenvalues and eigenvectors, which are to be obtained by feedback control. In this Note, a direct method is used for calculating the gain matrix, which achieves the desired eigenvalues exactly and the desired eigenvectors in a best least squares sense. Direct methods for eigenstructure assignment by least squares methods are described in numerous references.[1–6] Garrard et al.[5] and Low and Garrard[6] showed how to use handling quality specifications for helicopters to generate desired eigenstructures, which were then achieved in a least squares sense by feedback control. In this Note, the technique is extended to lateral-directional control of an aircraft. As an illustrative example, control of a tailless aircraft is considered. The method of state-space modeling of handling qualities could also be used in design of model matching H_2 or H_∞ controllers.

In a recent paper Mengali[7] developed a somewhat different approach to eigenstructure design in which he fixes a priori the components of the desired eigenvectors and then numerically minimizes a linear quadratic performance index to achieve the desired eigenvalues within specified limits. There are two major differences between the control design methodology in this Note and that of Mengali's. These are 1) the way in which the desired eigenstructure is selected and 2) the way in which the gain matrix is calculated. Mengali considers the control of a large transport. The open-loop response of this aircraft is relatively well behaved compared to the very unstable aircraft considered in this Note. It is not clear that Mengali's approach to closed-loop eigenvector selection would work well in the case of an extremely unstable aircraft. On the other hand, the method for eigenvector selection described in this Note could be used with Mengali's method for gain matrix calculation. This is an interesting idea because the direct method may, in some cases, result in large actuator deflections, whereas Mengali's method automatically trades off eigenvalue location (bandwidth) with surface activity. This tradeoff would have to be done by hand using the direct method.

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Design Technique

The system to be controlled is given in linear state-space form as

$$\dot{x} = Ax + Bu \tag{1}$$

and the control is a linear function of the state vector

$$u = -Kx \tag{2}$$

The feedback gain matrix is selected such that this control law results in desired placement of closed-loop eigenvalues and shaping of closed-loop eigenvectors. In the method presented, the desired eigenstructure is derived from a state-space model that represents the desired response. This model is given by

$$\dot{x}_d = A_d x_d + B_d x_c \tag{3}$$

The question is how to select A_d and B_d to give specified handling qualities. It is widely accepted that a first-order, roll-rate response and a second-order, side-slip response are desirable for acceptable handling qualities. Although many papers have developed controllers that decouple roll rate and side slip, it is clear from a knowledge of basic handling qualities that this is not desirable for stability augmentation because it implies neutral static lateral stability, i.e., L_β. (Neutral lateral stability may be desirable in a control augmentation system.) Also a stable, long time constant, spiral response is beneficial to good handling qualities.[8]

Desired lateral-directional response is modeled as

$$\begin{bmatrix} \dot\beta \\ \dot\phi \\ \dot p \\ \dot r \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{g\cos\theta}{V} & 0 & -1 \\ 0 & 0 & 1 & 0 \\ L_\beta & 0 & L_p & 0 \\ N_\beta & 0 & 0 & N_r \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -L_p & 0 \\ 0 & -N_\beta \end{bmatrix} \begin{bmatrix} p_c \\ \beta_c \end{bmatrix} \tag{4}$$

There is no cross coupling in the B_d matrix to avoid a roll-rate command as input to the yaw axis or vice versa. Because A_d determines the eigenstructure, it is specified so that it has a first-order roll subsidence mode, a second-order Dutch roll mode, a stable spiral mode, and static lateral stability. The characteristic equation for A_d is

$$(L_p - s)\left(-s^3 + N_r s^2 - N_\beta s\right) + \left(\frac{g\cos\theta}{V}\right)(N_r - s)L_\beta = 0 \tag{5}$$

If $g\cos\theta/V$ is small, then the nonzero roots of this equation are

$$s = L_p, \qquad s = (N_r/2) \pm \tfrac{1}{2}\sqrt{N_r - 4N_\beta} \tag{6}$$

Assuming s small to get the spiral root yields

$$\left(L_p N_\beta + L_\beta \frac{g\cos\theta}{V}\right)s - \left(\frac{g\cos\theta}{V}\right)N_r L_\beta = 0 \tag{7}$$

Substituting numerical values in these approximations gives

$$s = -4, \qquad s = -0.0389, \qquad s = -0.5 \pm j1.9365$$

The actual eigenvalues of A_d are

$$s = -4.0289, \qquad s = -0.0382, \qquad s = -0.4665 \pm j1.9581$$

The approximations just given are very similar to the three-degree-of-freedom Dutch roll approximation of McRuer et al.[9] for Y_β zero and if the derivatives are prime. It is easy to include Y_β in the formulation just given; however, in the flying quality specifications, no unambiguous specification of the desired value of this variable appears to exist and so it was set equal to zero for the sake of simplicity.